NASA TECH BRIEF

Self-Starting Procedure Simplifies Numerical Integration

The problem:

To devise a self-starting, multistep procedure for the numerical integration of ordinary differential equations. Most multistep methods are not self-starting, and single step methods such as that of Runge-Kutta are used to obtain starting values for the integration. This requires nonesssential tallying to determine enough starting values.

The solution:

The classical, multistep, predictor-corrector procedures for the numerical solution of systems of ordinary differential equations are generalized to provide compatible, self-starting methods that produce all the required backward differences directly from the initial equations. Explicit algorithms and tables of numerical coefficients are given for starting and continuing the numerical integration of the equations.

How it's done:

The general problem is to devise algorithms for calculating x_n, y_n, and f_n [where $x_n=x(t_n)$, $y_n=y(t_n)$, and $f_n=f(x_n, y_n, t_n)$] for $n=1, 2, 3, \ldots$, given the differential equations $dx/dt=y$ and $dy/dt=f(x, y, t)$ and the initial values $x_0=x(t_0)$ and $y_0=y(t_0)$. The theory for first order systems is obtained by ignoring x in these equations. The procedure used is the conventional one of approximating the function (f) by a polynomial t of degree q.

In order to achieve the best compromise between the requirements of speed, accuracy, and programming compactness, the following procedures are used in the integration:

I. Fourth order methods are used for first order equations ($q=4$) and sixth order methods are used for second order equations ($q=6$).

II. The iterated starter, which initializes the algorithms and then iterates the single set of equations, is used and iterated eight times. The iterated starter is superior to a "bootstrap" starter (essentially an efficient way of obtaining first approximations in the right hand side of the algorithms) because the "bootstrap" starter, although efficient in practice, is awkward and space-consuming when programmed for automatic computers due to the multiplicity of algorithms and matrices required.

III. The summed form of the predictor–corrector algorithm is used in backward-difference form. The effectiveness of the summed form of the predictor–corrector formula has long been known to astronomers. The use of backward differences in the forward integration is preferable to the use of backward ordinates for two reasons: (1) the backward ordinate formula tends to add nearly equal quantities of alternating sign, whereas the backward-difference formula adds monotonically decreasing quantities; and (2) the availability of the difference tables makes error estimation and automatic adjustment of the interval size a straightforward procedure.

IV. Four extra significant decimal digits are carried, in floating-point form, to control round-off errors.

Note:

Further information concerning this innovation is presented in NASA TN D-2936, "Self-Starting Multistep Methods for the Numerical Integration of Ordinary Differential Equations" by William A. Mersman, July 1965, available from the Clearinghouse for

(continued overleaf)

Federal Scientific and Technical Information, Springfield, Virginia 22151; price $2.00. Inquiries may also be directed to:

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Patent status:

No patent action is contemplated by NASA.

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